Near-Far Resistance of MC-DS-CDMA Communication Systems

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***Abstract*—In this paper, the near-far resistance of the minimum mean square error (MMSE) detector is derived for the multicarrier direct sequence code division multiple access (MC-DS-CDMA) communication systems. It is shown that MC-DS-CDMA has better performance on near-far resistance than that of DS-CDMA.**

***Index Terms*—Code division multiaccess, Near-far resistance, wireless communication**

I. Introduction

Multicarrier CDMA, which combines the Orthogonal Frequency Division Multiplexing (OFDM) based multicarrier transmissions and CDMA based multiuser access, is a promising technique for future 4G broadband multiuser communication systems. The application of OFDM greatly resolves the difficulty raised by multipath fading that is especially severe for broadband communication systems. On the other hand, the application of CDMA greatly simplifies the multi-access and synchronization design.

There have been many different types of multicarrier CDMA systems proposed [1][2]. One of them is MC-DS-CDMA [3], where each OFDM block (after IFFT and cyclic prefix) is block-wise spreaded, i.e., the OFDM block is spreaded into multiple OFDM blocks, each multiplied with different chip of the spreading code. A major feature of MC-DS-CDMA communication system is that each OFDM subcarrier works like DS-CDMA. Specifically, if there is only one subcarrier, then the MC-DS-CDMA reduces to a conventional DS-CDMA. One of the major advantages of MC-DS-CDMA is that each DS-CDMA signal (in each subcarrier) of a user can be maintained orthogonal to that of all the other users, when orthogonal spreading codes are used. As a result, multi-access interference (MAI) is mostly avoided, which may greatly enhance the performance over conventional DS-CDMA.

However, the well-known near-far problem in a multiuser setting still places fundamental limitations on the performance of MC-DS-CDMA communication systems. Therefore, near-far resistance remains one of the most important performance measures for MC-DS-CDMA systems. Furthermore, it would be interesting to explore whether the multicarrier has benefits on near-far resistance or not. It is worth mentioning that near-far resistance also depends on the type of the receiver detector. In this paper, the near-far resistance of the widely used MMSE detector is derived for the MC-DS-CDMA systems. It is shown that MC-DS-CDMA has better performance on near-far resistance than that of DS-CDMA systems.

II. System Model

Consider an asynchronous MC-DS-CDMA systems with N_c subcarriers and J active users in a multipath fading channel. Spreading codes of length L_c are used to distinguish different users.

It is shown in [4][5] that MC-DS-CDMA and DS-CDMA have the similar baseband MIMO system model. The total received signal vector $\chi_M(n)$ observed in additive white Gaussian noise **w**(n) can be expressed as [4][5].

$$\chi_N(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n) \qquad (1)$$

Where N is the smoothing factor, **H** is the signature matrix and **s**(n) is the transmitted information symbol vector. A smoothing factor N is indispensable to capture a complete desired user symbol since the detector does not know the staring time of each desired user symbol.

Since the channel effect on received energy can always be incorporated into a diagonal amplitude matrix **A**, without loss of generality, we assume that the columns of the channel matrix **H** are all normalized in the following sections. Then (1) can be rewritten as

$$\chi_N(n) = \mathbf{H}\mathbf{A}\mathbf{s}(n) + \mathbf{w}(n) \qquad (2)$$

III. Near-far resistance of MMSE receiver for MC-DS-CDMA systems

The following assumptions will be made throughout this paper. **AS1**: The symbols in **s**(n) are independently and identically distributed (i.i.d), with variance 1 (since symbol energy can also be absorbed into the diagonal matrix **A**). **AS2**: The noise is zero mean white Gaussian. **AS3**: The signature matrix **H** is of full column rank (known as the identifiability

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condition in the blind multiuser detection literatures). Note **AS3** is a reasonable assumption in practice considering the randomness of the multipath channels [6].

Without loss of generality, we assume that the dth symbol in **s**(n) is the desired transmitted symbol of the desired user and simply denote it by $s_d(n)$ (note the subscript d of $s_d(n)$ only represents its position in **s**(n)). Therefore, the MMSE detector weight vector is given by $\mathbf{f}_{mmse}=\mathbf{R}^{-1}\mathbf{H}_d$ [6] , where **R** is the autocorrelation matrix of the received signal $\chi_M(n)$ and $\mathbf{H_d}$ is the dth column in corresponding to the desired transmitted symbol $s_d(n)$. It is also well known that when noise approaches zero, the zero forcing (ZF) detector is proportional to the MMSE detector [6], $\mathbf{f}_{zf}= \alpha\mathbf{f}_{mmse}$, where α is a constant. Therefore, both detectors share the same near-far resistance. We have the following Proposition on near-far resistance of MC-DS-CDMA systems.

A. Proposition 1

Proposition 1 *The near-far resistance of the MMSE detector for the MC-DS-CDMA system (2) is* $\bar{\eta}_d=\dfrac{1}{(\mathbf{H}^H\mathbf{H})^{-1}_{(d,d)}}$ *, where the subscript (d, d) denotes choosing the element at the dth row and dth column.*

Proof: By applying the zero forcing detector to the received signal vector, the output contains only the useful signal and ambient Gaussian noise. The amplitude of the useful signal at the output is $\mathbf{f}_{zf}^H\mathbf{H}_d A_d s_d(n)$. Therefore, the energy of the useful signal at the output is $E_s=E[\mathbf{f}_{zf}^H\mathbf{H}_d A_d s_d(n) s_d^*(n) A_d \mathbf{H}_d^H \mathbf{f}_{zf}]=A_d^2\mathbf{f}_{zf}^H\mathbf{H}_d\mathbf{H}_d^H\mathbf{f}_{zf}$. The variance of the noise is $E_n=\sigma^2\mathbf{f}_{zf}^H\mathbf{f}_{zf}$ where σ^2 is the power spectral density of white Gaussian noise. Using the definition in [7], the asymptotic multiuser efficiency (AME) for the desired transmitted symbol is show below

$$\eta_d = \lim_{\sigma\to 0}\frac{\sigma^2[Q^{-1}(P_d(\sigma))]^2}{A_d^2} = \lim_{\sigma\to 0}\frac{\sigma^2 E_s/E_n}{A_d^2} = \lim_{\sigma\to 0}\frac{\sigma^2 A_d^2\mathbf{f}_{zf}^H\mathbf{H}_d\mathbf{H}_d^H\mathbf{f}_{zf}}{\sigma^2\mathbf{f}_{zf}^H\mathbf{f}_{zf}A_d^2}$$
$$=\lim_{\sigma\to 0}\frac{\mathbf{f}_{mmse}^H\mathbf{H}_d\mathbf{H}_d^H\mathbf{f}_{mmse}}{\mathbf{f}_{mmse}^H\mathbf{f}_{mmse}}=\lim_{\sigma\to 0}\frac{\mathbf{H}_d^H(\mathbf{H}\mathbf{A}^2\mathbf{H}^H+\sigma^2 I)^+\mathbf{H}_d\mathbf{H}_d^H(\mathbf{H}\mathbf{A}^2\mathbf{H}^H+\sigma^2\mathbf{I})^+\mathbf{H}_d}{\mathbf{H}_d^H(\mathbf{H}\mathbf{A}^2\mathbf{H}^H+\sigma^2\mathbf{I})^+(\mathbf{H}\mathbf{A}^2\mathbf{H}^H+\sigma^2\mathbf{I})^+\mathbf{H}_d}$$
$$=\frac{\mathbf{H}_d^H\mathbf{H}^{H^+}\mathbf{A}^{-2}\mathbf{H}^+\mathbf{H}_d\mathbf{H}_d^H\mathbf{H}^{H^+}\mathbf{A}^{-2}\mathbf{H}^+\mathbf{H}_d}{\mathbf{H}_d^H\mathbf{H}^{H^+}\mathbf{A}^{-2}\mathbf{H}^+\mathbf{H}^{H^+}\mathbf{A}^{-2}\mathbf{H}^+\mathbf{H}_d} \tag{3}$$
$$=\frac{A_d^{-4}}{\mathbf{H}_d^H\mathbf{H}^{H^+}\mathbf{A}^{-2}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{A}^{-2}\mathbf{H}^+\mathbf{H}_d}=\frac{1}{(\mathbf{H}^H\mathbf{H})^{-1}_{(d,d)}}$$

where $P_d(\sigma)=Q(\sqrt{\frac{E_s}{E_n}})$, Q is the complementary Gaussian cumulative distribution function, "+" represents pseudoinverse. In (3), we have used the facts that $(\mathbf{H}\mathbf{A}^2\mathbf{H}^H)^+=(\mathbf{H}^+)^H\mathbf{A}^{-2}\mathbf{H}^+$, $(\mathbf{H}^H\mathbf{H})^{-1}=\mathbf{H}^+(\mathbf{H}^+)^H$ as well as $\mathbf{H}^+\mathbf{H}_d=[0\cdots010\cdots0]^H$, where 1 is in the dth position. Note the first and last equalities are based on the assumption of full column rank of the channel matrix **H**. From (3), it is seen that the AME does not depend on the interfering signal amplitudes. Thus, it is equal to the near-far resistance $\bar{\eta}_d$ [7]. It then follows that $\bar{\eta}_d=\eta_d=\dfrac{1}{(\mathbf{H}^H\mathbf{H})^{-1}_{(d,d)}}$

Proposition 1 can be carried one step further to reach an expression that facilitates comparison of near-far resistance between multicarrier and single carrier DS-CDMA systems. Before proceeding further, however, we need to define some useful matrices. Let *I* denote the subspace spanned by interference signature vectors $\mathbf{H}_i$, $i\neq d$ where $\mathbf{H}_i$ denotes the ith column in the signature matrix **H**. $\tilde{\mathbf{H}}$ is the matrix obtained by deleting the dth column $\mathbf{H}_d$ from **H**. It is easy to show that $C(\tilde{\mathbf{H}})=\mathrm{I}$, where $C(\square)$ represents the column space. Denote $\mathbf{M}\square\mathbf{H}^H\mathbf{H}$, $\mathbf{R}_d=\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ and $r_d=\tilde{\mathbf{H}}^H\mathbf{H}_d$ is a vector resulting from deleting the dth entry from the dth column of **M**. Note $\mathbf{R}_d$ is non-singular due to **AS3**. We have the following proposition.

B. Proposition 2

Proposition 2 The near-far resistance in Proposition 1 can be rewritten as

$$\bar{\eta}_d=\frac{1}{(\mathbf{H}^H\mathbf{H})^{-1}_{(d,d)}}=1-\mathbf{r}_d^H\mathbf{R}_d^{-1}\mathbf{r}_d \tag{4}$$

Proof: Equation (3) can be rewritten as $\bar{\eta}_d=\dfrac{1}{(\mathbf{H}^H\mathbf{H})^{-1}_{(d,d)}}=\dfrac{\det(\mathbf{M})}{(-1)^{d+d}\det(\mathbf{R}_d)}$, where $\det(\square)$ represents the determinant of a matrix. To compute det(**M**), let i = d and do the following row and column operations on **M**: 1) exchange the ith row column with the (i+1)th column; 2) exchange the ith row with the (i+1)th row and set i = i + 1. If i ‘“ col(**H**), where col(**H**) denotes the number of columns of **H**, go to step 1; else terminate the row and column operations. We finally obtain $\bar{\mathbf{M}}=\begin{bmatrix}\mathbf{R}_d & \mathbf{r}_d\\ \mathbf{r}_d^H & 1\end{bmatrix}$. Since from **M** to $\bar{\mathbf{M}}$ only even number of exchange operations are executed, as a result we have $\det(\mathbf{M})=\det(\bar{\mathbf{M}})=\det(\mathbf{R}_d)-\mathbf{r}_d^H\mathbf{R}_d^{adj}\mathbf{r}_d$, where adj represents adjoint of a matrix and the second equality is resulted from a standard matrix equality [8] (pp. 50). Therefore, $\dfrac{\det(\mathbf{M})}{\det(\mathbf{R}_d)}=\dfrac{\det(\mathbf{R}_d)-\mathbf{r}_d^H\mathbf{R}_d^{adj}\mathbf{r}_d}{\det(\mathbf{R}_d)}=1-\mathbf{r}_d^H\mathbf{R}_d^{-1}\mathbf{r}_d$.

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IV. Comparison of Near-Far Resistance of MMSE Detector between Multicarrier and Single Carrier DS-CDMA Communication Systems

It is interesting to investigate how does the multicarrier scheme affect the near-far resistance in DS-CDMA systems. To this end, we analyze the near-far resistance of the MMSE detector for 1). DS-CDMA and 2). MC-DS-CDMA.

In order to facilitate fair comparison between different scenarios, we make the following assumption. **AS4**: the processing gain L_c, the system load J, the smoothing factor N, the distributions of multipath delay spread and asynchronous user delay are the same under different scenarios. Since the dimension of the signature matrix will prove to be useful for our following derivations, we now specify those parameters. Let $\mathbf{H}_1$ denote the signature matrix of the DS-CDMA system and $\mathbf{H}_2$ denote the signature matrix of MC-DS-CDMA systems. Under **AS4**, the dimensions of the signature matrix $\mathbf{H}_1$ and $\mathbf{H}_2$ are $NL_c \times J(L_h^{DS-CDMA}+N-1)$ and $NL_cN_c \times JN_c(L_h^{Mc-DS-CDMA}+N-1)$ respectively [5][9], where L_h (non-negative integer) is related to the maximum multipath delay spread and the maximum asynchronous user delay, and is defined in [5][9] as follows.

$$L_h^{DS-CDMA}=\max_j \left\lceil \frac{L_c+L_g-1+d_j}{L_c} \right\rceil \tag{5}$$

$$L_h^{MC-DS-CDMA} = \max_j \left\lceil \frac{N_cL_c+L_g-1+d_j}{N_cL_c} \right\rceil \tag{6}$$

where L_g denotes the maximum multipath delay spread and d_j denotes the jth user's asynchronous user delay.

Next we will compare the near-far resistance of MMSE detector under different scenarios. The value of the near-far resistance derived in Proposition 1 clearly depends on the multipath channels and asynchronous transmission delays. Since these parameters are random in nature, it is more meaningful to compare the statistical average of the near-far resistance rather than a particular random realization. To this end, we need an additional assumption. **AS5**: Under the ith scenario, assume $\mathbf{H}_d^i$ (the vector in $\mathbf{H}_i$ which is corresponding to the desired transmitted symbol of the desired user) is a random vector with a probability density function $\mathrm{N}_c(\mathbf{0}_{row(\mathbf{H}_i)}, \frac{1}{row(\mathbf{H}_i)}\mathbf{I}_{row(\mathbf{H}_i)})$ and is statistically independent of the interference subspace I (since it won't affect the derivation, here I is a general expression which includes all scenarios), where N_c represents the complex normal distribution, $row(\mathbf{H}_i)$ denotes the number of rows in $\mathbf{H}_i$, $\mathbf{0}_{row(\mathbf{H}_i)}$ represents the $row(\mathbf{H}_i)\times 1$ zero vector, and $\mathbf{I}_{row(\mathbf{H}_i)}$ represents the $row(\mathbf{H}_i)\times row(\mathbf{H}_i)$ identity matrix. The fact that the variance is $1/row(\mathbf{H}_i)$ is because $\mathbf{H}_d^i$ is normalized. We then have the following proposition.

A. Proposition 3

Proposition 3 *Denote $\bar{\bar{\eta}}_d^1$ as the expectation of the near-far resistance of the MMSE detector in DS-CDMA and $\bar{\bar{\eta}}_d^2$ as the expectation of the near-far resistance of the MMSE detector in MC-DS-CDMA. Then under* ***AS3***, ***AS4***, *and* ***AS5***, $\bar{\bar{\eta}}_d^1 < \bar{\bar{\eta}}_d^2$. *Proof:* under **AS3**, starting from (4), the conditional expectation of $\bar{\eta}_d^i$, conditioning on the interference subspace I, is given by

$$\begin{aligned} E[\bar{\eta}_d^i|I] &= 1-E[\mathbf{r}_d^{iH}(\mathbf{R}_d^i)^{-1}\mathbf{r}_d^i|I] = 1-E[tr\{\mathbf{r}_d^i\mathbf{r}_d^{iH}(\mathbf{R}_d^i)^{-1}\}|I] \\ &= 1-E[tr\{\tilde{\mathbf{H}}_i^H\mathbf{H}_d^i\mathbf{H}_d^{iH}\tilde{\mathbf{H}}_i(\mathbf{R}_d^i)^{-1}\}|I] \\ &= 1-E[tr\{\mathbf{H}_d^i\mathbf{H}_d^{iH}\tilde{\mathbf{H}}_i(\mathbf{R}_d^i)^{-1}\tilde{\mathbf{H}}_i^H\}|I] \\ &= 1-tr\{E[\mathbf{H}_d^i\mathbf{H}_d^{iH}|I]\tilde{\mathbf{H}}_i(\mathbf{R}_d^i)^{-1}\tilde{\mathbf{H}}_i^H\} \\ &= 1-\frac{1}{row(\mathbf{H}_i)}tr\{\tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i(\mathbf{R}_d^i)^{-1}\} \\ &= 1-\frac{1}{row(\mathbf{H}_i)}tr\{\mathbf{R}_d^i(\mathbf{R}_d^i)^{-1}\} \\ &= 1-\frac{col(\mathbf{H}_i)-1}{row(\mathbf{H}_i)} \end{aligned} \tag{7}$$

where $tr(\cdot)$ represents the trace of a matrix, $\tilde{\mathbf{H}}_i$, $\mathbf{r}_d^i$ and $\mathbf{R}_d^i$ are defined similar as in section 3 for the ith scenario. The sixth equality is based on the property of conditional expectation and seventh equality is based on the fact that $E[\mathbf{H}_d^i\mathbf{H}_d^{iH}|I]=E[\mathbf{H}_d^i\mathbf{H}_d^{iH}]=\frac{1}{row(\mathbf{H}_i)}\mathbf{I}$ due to **AS5**. Based on (5) (6) and **AS4**, it is straightforward to show that

$$E[\bar{\eta}_d^1|I]=1-\frac{J(L_h^{DS-CDMA}+N-1)-1}{NL_c} \tag{8}$$

$$E[\bar{\eta}_d^2 \mid I] = 1-\frac{JN_c(L_h^{MC-DS-CDMA}+N-1)-1}{NL_cN_c} \tag{9}$$

Subtracting (8) from (9), we have

$$\begin{aligned} &E[\bar{\eta}_d^2 \mid I]-E[\bar{\eta}_d^1 \mid I] \\ &= \frac{J(L_h^{DS-CDMA}+N-1)-1}{NL_c}-\frac{JN_c(L_h^{MC-DS-CDMA}+N-1)-1}{NL_cN_c} \\ &= \frac{JN_c(L_h^{DS-CDMA}-L_h^{MC-DS-CDMA})+1-N_c}{NL_cN_c} \end{aligned} \tag{10}$$

Under **AS4**, J, N and L_c are the same under different scenarios for each realization of I. It is also easy to show that $\left\lceil \frac{x}{a} \right\rceil > \left\lceil \frac{x}{ab} \right\rceil$ when x > a and b > 1. In other word, $L_h^{DS-CDMA} > L_h^{MC-DS-CDMA}$. Based on (10), we have $E[\bar{\eta}_d^1|I] < E[\bar{\eta}_d^2|I]$ for each realization of I.

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Since $\bar{\bar{\eta}}_d^i = E_I[E[\bar{\eta}_d^i | I]]$, i = 1, 2, the claims in Proposition 3 are proven. Note when $N_c = 1$, MC-DS-CDMA reduces to a conventional DS-CDMA and thus has the same near-far resistance as DS-CDMA.

Conclusions

The well-known near-far problem in a multiuser setting still places fundamental limitations on the performance of CDMA communication systems. In this paper, the near-far resistance of the MMSE detector is derived for the MC-DS-CDMA systems and compared with DS-CDMA. It is shown that MC-DS-CDMA has better performance on near-far resistance than that of DS-CDMA.

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